Research Article

Application of the Handshaking Lemma in the Dyeing Theory of Graph

Roland Forson*[1], Cai Guanghui[1], Richmond Nii Okle[1], Daniel Ofori Kusi[2], Elise Hamunyela[3]

[1]Department of Statistics and Mathematics, Zhejiang Gongshang University

[2]Department of Mathematics, University of Free State, South Africa

[3]Department of Mathematics, University of Namibia

*Corresponding Author: Roland Forson, Department of Statistics and Mathematics, Zhejiang Gongshang University, China, E-mail: forson@pop.zjgsu.edu.cn

Abstract

The handshaking lemma is one of the important branches of graph theory. The content is widely applied in topology and computer science. The basis of the development of the dyeing theory used in this research paper is to discuss the application of the right transfer method in dyeing theory.

Keywords: Right transfer method; Four-color conjecture; Dyeing graph theory; Euler; Handshaking lemma

1. Introduction

The four-color problem forms the famous four-color conjecture. The four-color conjecture is available for any flat-picture graph   such that, $\chi * (G) \leq 4$. The four-color lemma has seen extensive research in graph theory, but the problem is $NP - hard$ and computer results proved convincing in edge-dyed [2], surface-dyed [3, 4] , and color-dyed [1, 8] with a restricted strip. In the literature above, the methods of proof commonly used in mathematics, such as the traditional direct proof method, the reverse proof method, the mathematical induction method, etc., and the graph theory proves its unique method and technology, such as the shortest (long) path method, and the maximum edge method [1, 5, 8].

2. The Development of Dyeing Theory

The process of applying the weight transfer method is based on the handshake lemma. Let G be a connected plane graph, then according to Euler theorem;

$$V - \varepsilon + \phi = 2$$

Where   is the number of vertices of G, the number of edges of G is ε , and the number of planes of G is ϕ . The handshake lemma [2, 5, 9] sets G as a communication flat graph, and that,

$$\sum_{v \in V(G)} d_G(v) = \sum_{f \in F(G)} d_G(f) = 2\varepsilon$$

Where $F(G)$ is the face set of G.

If we set G as a connected flat chart, for any real number $k, l > 0$; following constant equation is established:

$$\sum_{v \in V(G)} (kd_G(v) - 2(k+l)) + \sum_{f \in F(G)} (ld_G(f) - 2(k+l)) = -4(k+l)$$

3. Power Transfer Method

Applying Euler Formula $V - \varepsilon + \phi = 2$ and handshaking lemma, $\sum_{v \in V(G)} d_G(v) = \sum_{f \in F(G)} d_G(f) = 2\varepsilon$ explains the

sum of the initial rights as a constant. By using the nonexistence of reducible sub-graphs, it is shown that the total number of new weight sums is different from the total number of initial weight sums [2, 10], and thus obtains the contradiction. It is proven that such a minimal counterexample of G does not exist [12, 13], so the graph belonging to C has the *P* attribute.

For any, $x, y \in V(G) \cup F(G)$, We remember $w(x \to y)$ denotes the weight transferred from *x* to *y*. To begin with, every element in $V(G) \cup F(G)$, the graph G remains unchanged; $(2)V(G) \cup F(G)$ the new weight of each element is not negative, which contradicts the Euler formula. In practice, this is often the case:

Yes, arbitrary $x \in V(G) \cup F(G)$ defines graph G as follows:

$x \in V(G)$, make $x \in V(G)$, $\forall f \in F(G) \, \exists \omega(f) = ld_G(f) - 2(l+k)$.

So $\sum_{v \in V(G) \cup F(G)} \omega(x) = -2(k+l)$.

This follows that, $x \in V(G) \cup F(G)$. The weights of each element are adjusted to $\omega'(x)$. Since it is only transferred within the elements, the addition and retention of the graph remains the same, that is,

$$\sum_{v \in V(G) \cup F(G)} \omega'(x) = \sum_{v \in V(G) \cup F(G)} \omega(x) < 0.$$ Furthermore we define an appropriate weight transfer rule, and verify that

each conforms to a certain rule structure, such as the result obtained $\sum_{v \in V(G) \cup F(G)} \omega'(x) \geq 0,$ which is a contradiction,

and the conclusion is proved.

Below 4 cases:

We give an example to illustrate the application of weight transfer method $[7, 9]$. Let G be a simple plan, and $\delta(G) \geq 2,$ if $g(G) \geq 7$, Then one of the following conditions holds:

(I) G contains a 2-point adjacent to a 2-point;

(II) G contains a 3-point adjacent to a 2-point and a $\leq$ 3-point;

(III) G contains a d-point adjacent to (d -1) 2-points, where $d \geq 4$

(a) (b) (c)
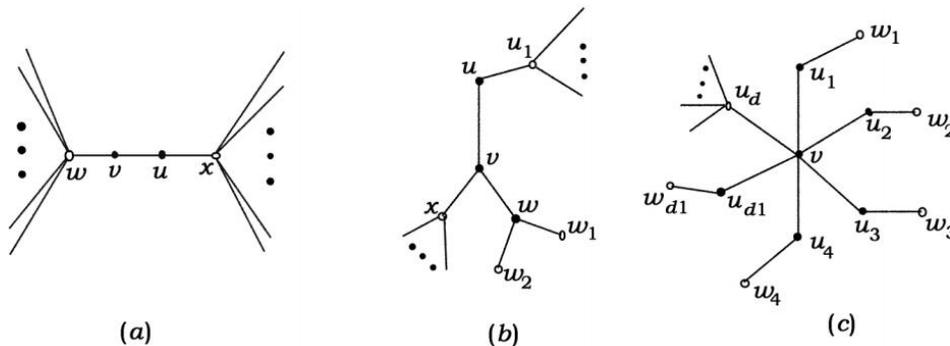
It is proved that G is the least reverse example to meet the example condition, and the Euler defined formula $V - \varepsilon + \phi = 2$ can be rewritten into the following form;

$$\sum_{v \in V(G)} (5d(v) - 14) + \sum_{f \in F(G)} 2d(f) - 14 = -28$$

Then, $\forall x \in V(G) \cup F(G)$ defines the initial weight function $\omega(x) : v \in V(G), \omega(v) = 5d(v) - 14$ provided that, $f \in F(G), \omega(f) = 2d(f) - 14$ and $\sum_{x \in V(G) \cup F(G)} \omega(x) = -28 < 0$ The transfer of power method is then used to obtain

a new number of ω' , where the rules for transfer of power are as follows:

$(R1)$: Transfer from each ≥ 3 point to each of its adjacent 2-points is 2

$(R2)$: Transfer from each ≥ 4 point to each of its adjacent vertex v, if $d(v) = 2$, then there is a transfer 2 to $d(v) = 3, from1$

we proved that, $\forall x \in V(G) \cup F(G)$, $\exists, \omega'(x) \geq 0$, and that is a contradiction. The set $f(G)$ is such that, $d(f) \geq 7$ and $\omega'(f) = \omega(f) \geq 0$.

Set v is any vertex of Figure G, if $d(v) = 2$, the $N_v(G) \geq 3$ and $\omega'(v) \geq \omega(v) + 2 \cdot 2 = 0$ such that, $d(v) = 3$, is attached to the $N_v(G)$. Thus all the $N_v(G) \geq 4$ is $w'(v) \geq \omega(v) + \min\{0, -2 + 1 \cdot 2\} = 1 > 0$ and that $d(v) \geq 4, \omega'(v) \geq \omega(v) - (d(v) - 2) \cdot 2 - 1 \cdot 2 = 3d(v) - 12 \geq 0$

Thus, $\omega'(x) \geq 0$, as proved.

References

1. West DB. Introduction to graph theory. Upper Saddle River, NJ: Prentice hall 2 (1996).

2. Gross JL, Yellen J (Eds.). Handbook of graph theory. CRC press (2004).

3. Bondy JA, Murty USR. Graph theory with applications. London: Macmillan 290 (1976).

4. Godsil C, Royle GF. Algebraic graph theory. Springer Science and Business Media 207 (2013).

5. Bollobás B. Modern graph theory. Springer Science and Business Media 184 (2013).

6. Biggs N, Biggs NL, Norman B. Algebraic graph theory. Cambridge university press 67 (1993).

7. Golumbic MC. Algorithmic graph theory and perfect graphs. Elsevier 57 (2004).

8. Forson R. Hamiltonicity of Total Domination 3-Connected Edge Critical Graph. Available at SSRN 2941978 (2017).

9. Gross JL, Tucker TW. Topological graph theory. Courier Corporation (2001).

10. Forson R, Guanghui C, Ofori S, et al. The Distribution Properties of Two-Parameter Exponential Distribution Order Statistics.

11. Gibbons A. Algorithmic graph theory. Cambridge university press (1985).

12. Golumbic MC. Algorithmic graph theory and perfect graphs. Elsevier 57 (2004).

13. Forson R, Nweit OT. A Generalization of Probabilistic Cauchy-Schwarz Inequality. Available at SSRN3080064 (2017).